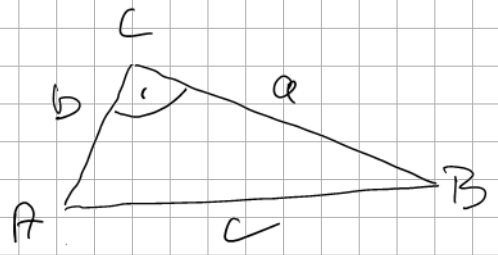
Vorkurs 28.2.2025

6. Trigonometrie

6.1 Trigonometrische Berechnungen

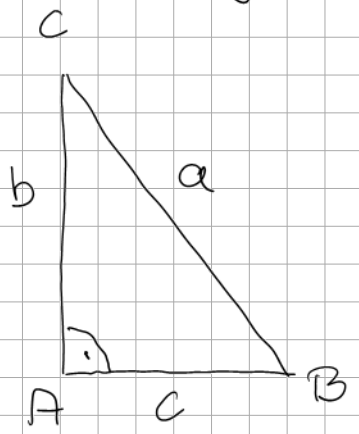
Rechtwinklige Dreiecke

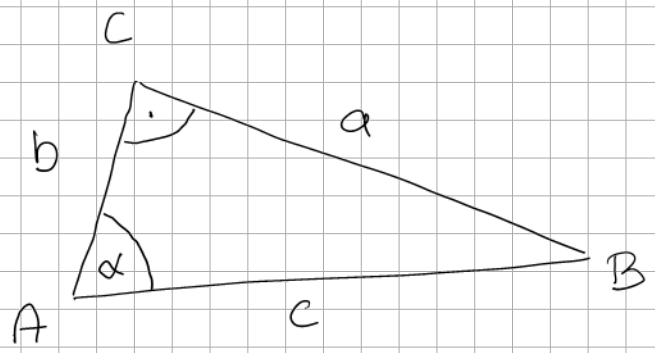
Pythagoras $a^2 + b^2 = c^2$



„ 1. Kathete^2 + 2. Kathete^2 = Hypotenuse^2 "

$b^2 + c^2 = a^2$





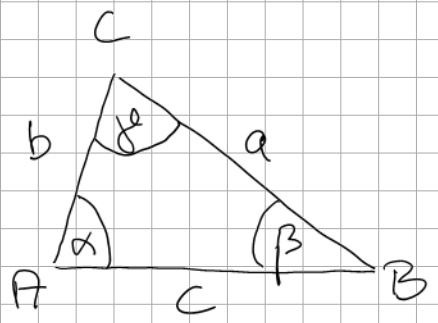
$$\sin(\alpha) = \frac{a}{c} \qquad \frac{\text{Gegenkathete}}{\text{Hypotenuse}}$$

$$\cos(\alpha) = \frac{b}{c} \qquad \frac{\text{Ankathete}}{\text{Hypotenuse}}$$

$$\tan(\alpha) = \frac{a}{b} \qquad \frac{\text{Gegenkathete}}{\text{Ankathete}}$$

$$\cot(\alpha) = \frac{b}{a} \qquad \frac{\text{Ankathete}}{\text{Gegenkathete}}$$

Allgemeines Dreieck



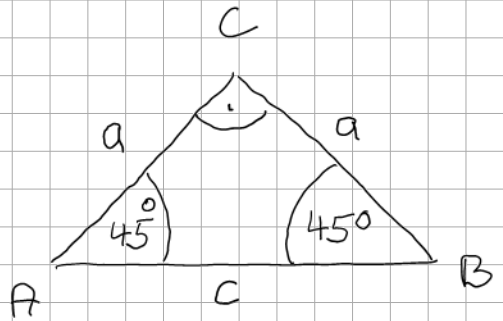
Sinussatz $\frac{a}{\sin(\alpha)} = \frac{b}{\sin(\beta)} = \frac{c}{\sin(\gamma)}$

Cosinussatz

$$a^2 = b^2 + c^2 - 2bc \cdot \cos(\alpha)$$
$$b^2 = a^2 + c^2 - 2ac \cdot \cos(\beta)$$
$$c^2 = a^2 + b^2 - 2ab \cdot \cos(\gamma)$$

Beispiele

1) $\sin(\alpha)$, $\cos(\alpha)$, $\tan(\alpha)$, $\cot(\alpha)$ für $\alpha = 45°$



$$c^2 = a^2 + a^2$$
$$c^2 = 2a^2$$
$$c = a\sqrt{2}$$

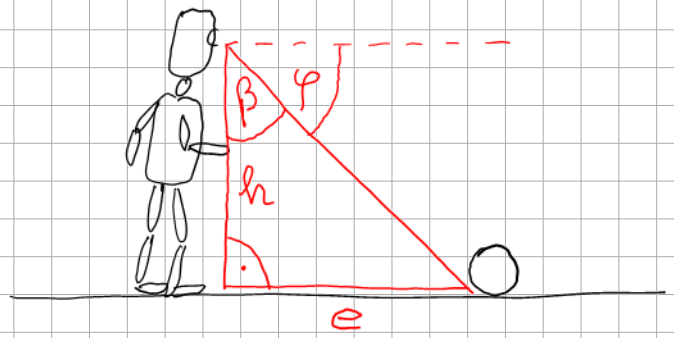
$$\sin(\alpha) = \frac{a}{c} = \frac{a}{a\sqrt{2}} = \frac{1}{\sqrt{2}} = \frac{1}{2}\sqrt{2}$$

$$\cos(\alpha) = \frac{a}{c} = \frac{a}{a\sqrt{2}} = \frac{1}{2}\sqrt{2}$$

$$\tan(\alpha) = \frac{a}{a} = 1$$

$$\cot(\alpha) = \frac{a}{a} = 1$$

2) Robotersteuerung (Robo-Cup)



Augenhöhe 28 cm $h = 28\,cm$

Neigungswinkel 40° $\varphi = 40°$

Wie weit ist der Ball entfernt? e

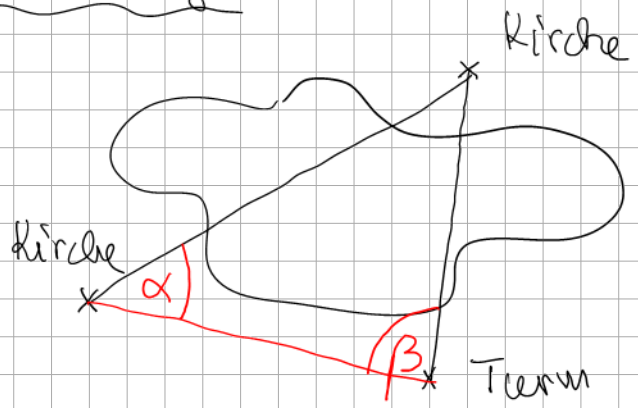
$\beta = 90° - \varphi$ $\quad \tan(\beta) = \frac{e}{h}$

$\beta = 50°$ $\quad e = h \cdot \tan(\beta)$

$e = 28 \cdot \tan(50°)$

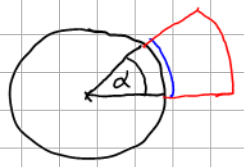
$e \approx 33{,}4\,cm$

3) Landvermessung



6.2 Bogenmaß

Winkel: 1) Grad 2) Bogenmaß



Bogenmaß: Bogenlänge im Verhältnis zum Radius

$\hat{=}$ Bogenlänge eines Kreises mit $r = 1$

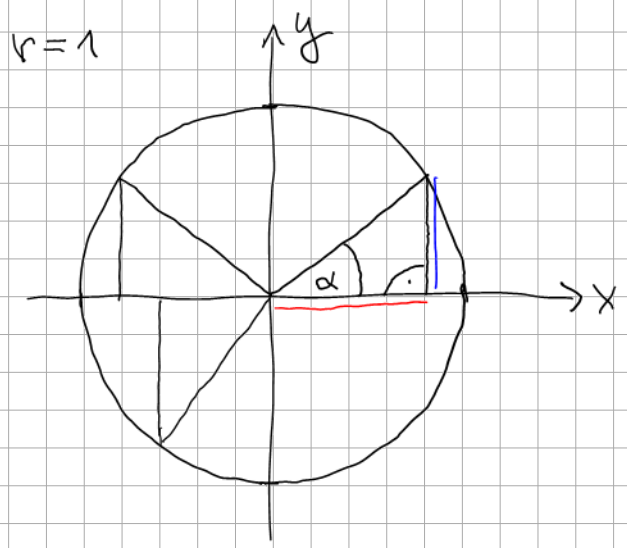
Vollwinkel $360°$ $\hat{=}$ Bogenlänge ist der Umfang des Kreises $2\pi r$

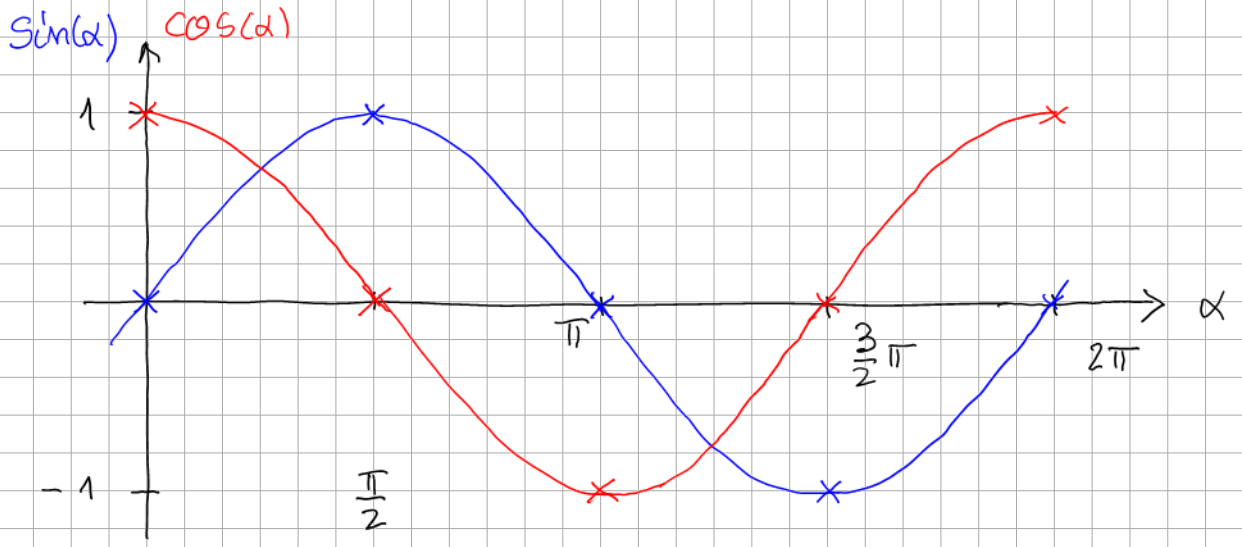
Bogenmaß $b = \frac{2\pi r}{r} = 2\pi$

α	$360°$	$180°$	$90°$	$270°$	$45°$	$30°$	$60°$	$1°$
b	2π	π	$\frac{\pi}{2}$	$\frac{3}{2}\pi$	$\frac{\pi}{4}$	$\frac{\pi}{6}$	$\frac{\pi}{3}$	$\frac{\pi}{180}$

$$b = \frac{\alpha}{360°} \cdot 2\pi$$

6.3 Winkelfunktionen, trigonometrische Funktionen



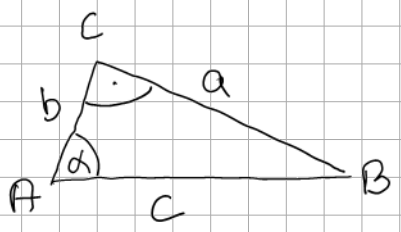


Periodische Funktionen $\sin(\alpha) = \sin(\alpha + k \cdot \pi)$, $k \in \mathbb{Z}$

$\cos(\alpha) = \cos(\alpha + k \cdot \pi)$

Zusammenhang $\sin(\alpha) = \cos(\alpha - \frac{\pi}{2})$

$\sin^2(\alpha) + \cos^2(\alpha) = 1$ $\qquad \sin^2(\alpha) = (\sin(\alpha))^2$

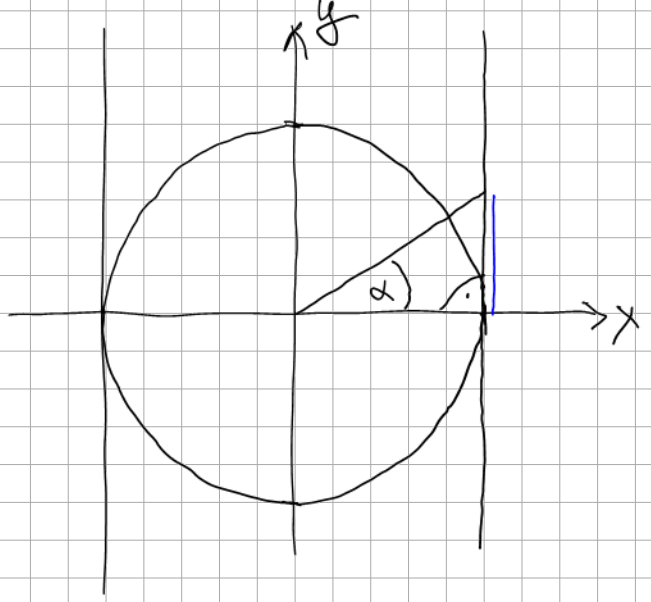


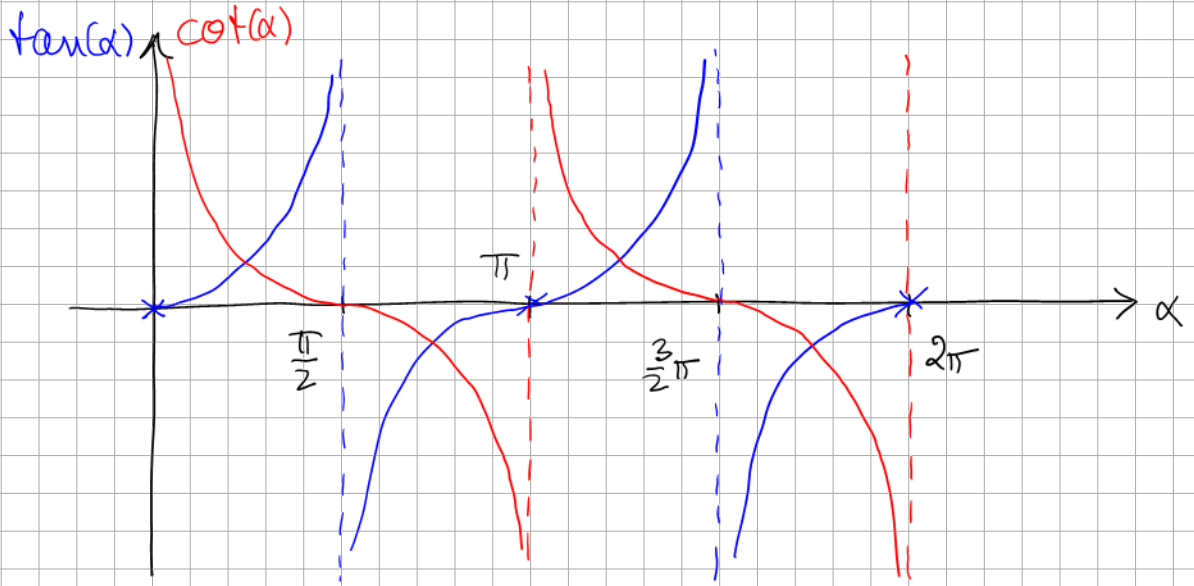
$\sin(\alpha) = \frac{a}{c}$ $\qquad \cos(\alpha) = \frac{b}{c}$

$$\sin^2(\alpha) + \cos^2(\alpha) = \left(\frac{a}{c}\right)^2 + \left(\frac{b}{c}\right)^2 = \frac{a^2}{c^2} + \frac{b^2}{c^2} = \frac{a^2+b^2}{c^2} = \frac{c^2}{c^2} = 1$$

Additionstheoreme

$$\sin(\alpha \pm \beta) = \sin(\alpha)\cos(\beta) \pm \sin(\beta)\cos(\alpha)$$
$$\cos(\alpha \pm \beta) = \cos(\alpha)\cos(\beta) \mp \sin(\alpha)\sin(\beta)$$





Periodische Funktionen

$$\tan(\alpha) = \tan(\alpha + k \cdot \pi), \quad k \in \mathbb{Z}$$
$$\cot(\alpha) = \cot(\alpha + k \cdot \pi)$$

Definitionslücken

$\tan(\alpha)$ ist nicht definiert für $\alpha = \frac{\pi}{2} + k \cdot \pi = \frac{2k+1}{2}\pi \quad k \in \mathbb{Z}$

$\cot(\alpha)$ ist nicht definiert für $\alpha = k \cdot \pi$

Zusammenhang zwischen Winkelfunktionen

$$\tan(\alpha) = \frac{\sin(\alpha)}{\cos(\alpha)} \qquad \cot(\alpha) = \frac{\cos(\alpha)}{\sin(\alpha)}$$

Aufgaben

Skript: Nr. 41 - 49

Nr. 42, 43, 44 Kantenlänge a